Dissipative quantum tunneling of two-level systems driven by dc-ac fields

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The time-dependent average population in a two-level system interacting with an Ohmic boson bath and driven by a dc-ac field is obtained. The study is performed within the noninteracting-blip approximation and for the high-frequency driving case. The integro-differential master equation and the exact formal solution for the average transition probability are obtained. The dissipative quantum decay and the transition temperature from the coherent to the incoherent motion are investigated. The effect of the external electric field on the the dissipative two-level system is found to destroy its coherence. [S1063-651X(98)05908-X]

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Recently, there has been much interest in understanding the effect of dissipation and finite temperature on the quantum mechanical tunneling in two-level systems [1,2]. Both Ohmic dissipation (which is relevant to the problem of macroscopic quantum tunneling) and strong electric field effects have received considerable attention [1–9] An exact path-integral solution for the average population of a dissipative two-level system was obtained [4]. The transient two-level system dynamics for the high-frequency driven case was studied by means of the series method defined by the recursion relations [5] and the integro-differential kinetic equation method [6,7]. Investigations based on the non-Markovian master equation approach [which is valid beyond the noninteracting-blip approximation (NIBA)] that governs the dynamics of two-level systems were also performed [8].

We address in this work the problem of the dissipative quantum tunneling in two-level systems driven by dc-ac fields. The evolution of the population in a given well is calculated with the integro-differential kinetic equation method [6,10,11], which was already used to study a host of phenomena including electron localization [10], low-frequency, and even harmonic generation [11,12], and the transient dynamics in the low-temperature limit of dissipative two-level systems under the influence of a external electric field [6,7]. In this paper the exact formal solution of the kinetic equations in the NIBA is obtained, which gives the average two-level system population when the high-frequency driving case is considered. We describe the dissipative relaxation of the system in the overdamped regime and we calculate the transition temperature in which the transition from the coherent to the incoherent motion occurs.

According to previous works [1,2], a two-level system driven by a dc-ac field $V(t)=\mu E_0+\mu E\cos\omega_0 t$ and in contact with a bosonic oscillator bath can be described by the spin-boson Hamiltonian

$$H_0=-\hbar\sigma_x\Delta/2+\hbar\sum_l \omega_l b_l^\dagger b_l+\sigma_z\sum_l g_l(b_l^\dagger+b_l)+\sigma_z V(t). \tag{1}$$

In the above equation σ_x and σ_z are the Pauli matrices; Δ corresponds to the tunneling matrix element between the two minima, which is related to the energy levels splitting; $b_l^\dagger$ and b_l are the creation and annihilation boson (phonon) operators, respectively; ω_l is the frequency of the lth boson; and g_l is the matrix element of the particle-boson coupling. In the driving field potential $V(t)$, E_0 is the dc electric field intensity, which breaks the symmetry of the two-level system; E is the amplitude of the laser field; μ is the transition dipole between the two energy levels; and ω_0 is the frequency of the driving laser field.

All necessary information on the role of the environment is contained in the spectral density function $J(\omega)$, which for the case of Ohmic dissipation is given by [1,2]

$$J(\omega)=(2\pi\hbar/q_0^2)\alpha\omega\exp(-\omega/\omega_c), \tag{2}$$

where q_0 is the the distance between the wells (a discussion of the connection between a double-well and a two-level model is presented in Ref. [1]); ω_c is the cutoff frequency, which is much larger than Δ ($\Delta/\omega_c\ll 1$ is the limit of primary interest for the macroscopic quantum coherence problem); and α is a dimensionless phenomenological parameter related to the dissipation [1,2].

In the high-frequency driven case, the oscillating driving field is effective in reducing the blip length, as stated by Grifoni *et al.* [5]. Thus, whenever the NIBA is applicable in the absence of a time-periodic field, it is justified even better in the presence of a high-frequency driving field for the Ohmic case [5]. In both the high-frequency driving field case ($\Delta/\omega_0\ll 1$) and in the long-time asymptotic limit, we obtain the master equation for the average population $\langle x(t)\rangle$ within the NIBA (for details, see Refs. [6,7,13]; hereafter, we take $\hbar=1$)

$$\frac{d\langle x(t)\rangle}{dt}=-\int_0^t[h(t-t_1)+g(t-t_1)\langle x(t_1)\rangle]dt_1, \tag{3}$$

with $\langle x(0)\rangle=1$ as the initial condition. The functions $h(t)$ and $g(t)$ are defined by

$$h(t)=\Delta^2 J_0[2a\ \sin(\omega_0 t/2)]\sin[b\omega_0 t]\times\sin[Q_1(t)/\pi]\exp[-Q_2(t)/\pi], \quad (4)$$

$$g(t)=\Delta^2 J_0[2a\ \sin(\omega_0 t/2)]\cos[b\omega_0 t]\times\cos[Q_1(t)/\pi]\exp[-Q_2(t)/\pi], \quad (5)$$

where

$$Q_1(t)\equiv\int_0^\infty \frac{J(\omega)}{\omega^2}\sin\ \omega t\ d\omega, \quad (6)$$

$$Q_2(t)\equiv\int_0^\infty \frac{J(\omega)}{\omega^2}(1-\cos\ \omega t)\coth\left(\frac{1}{2}\beta\omega\right)d\omega. \quad (7)$$

In the above expressions J_0 is the zeroth-order Bessel function, $a=2\mu E/\omega_0$, $b=2\mu E_0/\omega_0$, and $\beta=1/k_BT$. Through a Laplace transformation with respect to time, the exact formal solution for $x(\lambda)$ [the Laplace transform of $\langle x(t)\rangle$] within the NIBA is

$$x(\lambda)=\frac{1-h(\lambda)/\lambda}{\lambda+g(\lambda)}, \quad (8)$$

where $h(\lambda)$ and $g(\lambda)$ are, respectively, the Laplace transforms of $h(t)$ and $g(t)$. For $\alpha>1/2$, the functions $g(\lambda)$ and $h(\lambda)$ have well-defined expansions around zero,

$$g(\lambda)=g_0+\lambda g_1+\cdots, \quad (9)$$

$$h(\lambda)=h_0+\lambda h_1+\cdots, \quad (10)$$

with $g_1,h_1\ll 1$. Then, by applying an inverse Laplace transformation, we get

$$x(\tau)=X_0+[1-X_0]e^{-\tau\Gamma}, \quad (11)$$

where $\tau=\omega_0 t$. The relaxation rate Γ and the equilibrium population X_0 constants are given, respectively, by

$$\Gamma=\frac{g_0}{\omega_0}=\epsilon^2\delta^{2\alpha}\sqrt{\pi}\frac{\tilde{T}^{1-2\alpha}}{2\Gamma(\alpha)\Gamma(\alpha+\frac{1}{2})}\sum_{m=-\infty}^{\infty}J_m^2(a)\times\cosh[\pi(m+b)\tilde{T}]|\Gamma[\alpha+i(m+b)\tilde{T}]|^2, \quad (12)$$

$$X_0=-\frac{h_0}{g_0}=-\frac{\sum_{m=-\infty}^{\infty}J_m^2(a)\sinh[\pi(m+b)\tilde{T}]|\Gamma[\alpha+i(m+b)\tilde{T}]|^2}{\sum_{m=-\infty}^{\infty}J_m^2(a)\cosh[\pi(m+b)\tilde{T}]|\Gamma[\alpha+i(m+b)\tilde{T}]|^2}, \quad (13)$$

where $J_n(z)$ and $\Gamma(z)$ are, respectively, the nth Bessel and the Euler Gamma function and we have defined $\epsilon=\Delta/\omega_0$, $\delta=\omega_0/\omega_c$, and $\tilde{T}=\omega_0/\pi k_BT$.

According to Eq. (11), $x(\tau)$ presents an exponential incoherent relaxation to the equilibrium population value X_0 with a characteristic relaxation rate constant Γ. When $\tau\to\infty$, the average population is given approximately by $X_0=-h_0/g_0$, the equilibrium population. Both the equilibrium population X_0 and the relaxation rate Γ constant depend on the field parameters a and b, the dimensionless dissipation parameter α, and the system temperature T.

Figure 1 shows that the absolute value of the equilibrium population constant X_0 decreases when the system temperature is higher and/or the driving laser frequency ω_0 is small. The decreasing rate of X_0 is stronger when the field parameter a is small, which means a weaker dc field intensity E_0 and/or a higher driving laser frequency ω_0. On the other hand, Fig. 2 shows that the modified rate constant $\Gamma^*=\ln[2\Gamma/\epsilon^2\delta^{2\alpha}]$ is bigger when the field parameter increases, but smaller if the temperature of the system is higher. However, the modified rate constant Γ^* never vanishes. When the dimensionless dissipation parameter $\alpha>1/2$, we obtain that only the quantum dissipative relaxation can be observed in our system. So we can conclude that the localization phenomenon observed in the undriven case cannot be observed in the driven case [1,2].

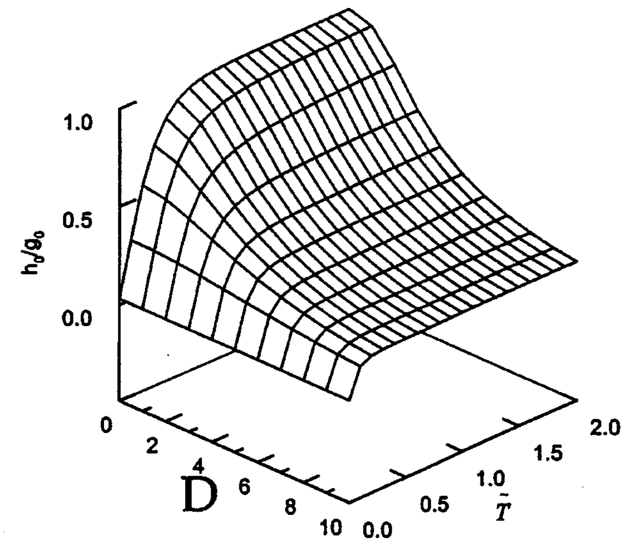

FIG. 1. Dependence of the absolute value of the equilibrium population constant h_0/g_0 on the scaled ac field $a=2\mu E/\omega_0$ and on the modified temperature $\tilde{T}$ ($\equiv\omega_0/\pi k_BT$). The figure is obtained by taking $b=2\mu E_0/\omega_0=1$ and $\alpha=1.3$.

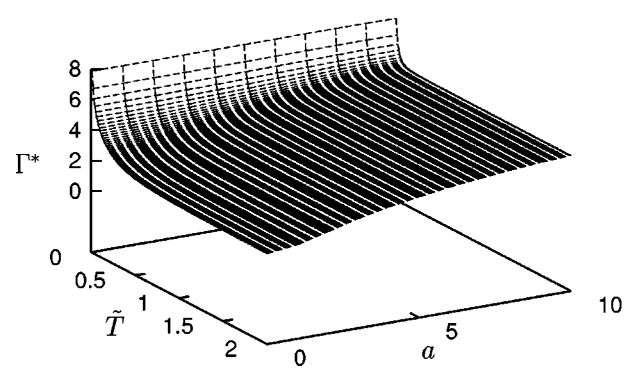

FIG. 2. Dependence of the modified rate constant on the intensity of ac field a and on the scaled temperature $\tilde{T}=\omega_0/\pi k_BT$. The figure is obtained by taking $b=2\mu E_0/\omega_0=1$ and $\alpha=1.3$. Γ^* stands for $\ln[2\Gamma/\epsilon^2\delta^{2\alpha}]$.

In the context of the macroscopic quantum coherence, the observation or not of an oscillatory behavior would be of fundamental significance to our understanding of quantum mechanics. Previous analyses on the role of the dimensionless dissipation parameter $\alpha > 1/2$ at $T=0$ have highlighted the absence of a ''coherent'' behavior when $\alpha > 1/2$ and that coherence is very unlikely to occur even for $T>0$ [1,6,13,14]. Consequently, we shall restrict ourselves in the following to the range of values $0<\alpha<1/2$. To investigate the transition from the the coherent to the incoherent motion, we limit ourselves to the low-temperature limit $\gamma \equiv \pi k_B T/\omega_c \ll 1$. In this case, it is straightforward to show that

$$h(\lambda)=\omega_0 \epsilon^2 \delta^{2\alpha} f_1(a,N,\alpha), \tag{14}$$

$$g(\lambda)=\gamma^{2\alpha-1}\frac{\sqrt{\pi}}{2\omega_c}\frac{\Delta^2\Gamma(1-\alpha)\Gamma(\lambda/2\gamma\omega_c+\alpha)}{\Gamma(\alpha+1/2)\Gamma(\lambda/2\gamma\omega_c-\alpha+1)}J_N^2(a) + \omega_0\epsilon^2\delta^{2\alpha} f_2(a,N,\alpha). \tag{15}$$

The functions $f_1(a,N,\alpha)$ and $f_2(a,N,\alpha)$ are given by

$$f_1(a,N,\alpha)=\frac{\pi}{2\Gamma(2\alpha)}\left\{\sum_{m=0}^{\infty} J_m^2(a)(m+N)^{2\alpha-1} + \sum_{m=1}^{\infty}{}' J_m^2(a)|N-m|^{2\alpha-1}\frac{N-m}{|N-m|}\right\}, \tag{16}$$

$$f_2(a,N,\alpha)=\frac{\pi}{2\Gamma(2\alpha)}\left\{\sum_{m=0}^{\infty} J_m^2(a)(m+N)^{2\alpha-1} + \sum_{m=1}^{\infty}{}' J_m^2(a)|N-m|^{2\alpha-1}\right\}, \tag{17}$$

where Γ is Euler's Gamma function and the prime to the summation means the exclusion of the term $m-N=0$.

Since no coherent behavior emerges whenever $b=2\mu E_0/\omega_0 \neq N$ [13], we set the scaled dc field strength to be an integer $b=2\mu E_0/\omega_0=N$. When the temperature is not zero, there is no branch point in $x(\lambda)$. However, the branch point ($\lambda=0$, when $T=0$, and $0<\alpha<1/2$) cuts degenerates into a set of poles on the negative real λ axis with an uneven spacing that grows roughly linearly with T. The complex-conjugate pair of poles moves towards the negative real axis, eventually hits it, and then moves along it towards opposite directions. If we now define $T^*(\alpha)$ as the temperature at which these two poles coincide [1,14], it is clear that (i) when $T<T^*(\alpha)$, a coherent motion, i.e., a damping oscillatory behavior, should be observed in the system and (ii) when $T>T^*(\alpha)$, the average population $\langle x(t)\rangle$ is given by a sum of decay exponentials and cannot present an oscillatory behavior.

We can calculate the transition temperature $T^*(\alpha)$ at which the transition from the coherent to the incoherent behavior occurs through [1,6,13,14]

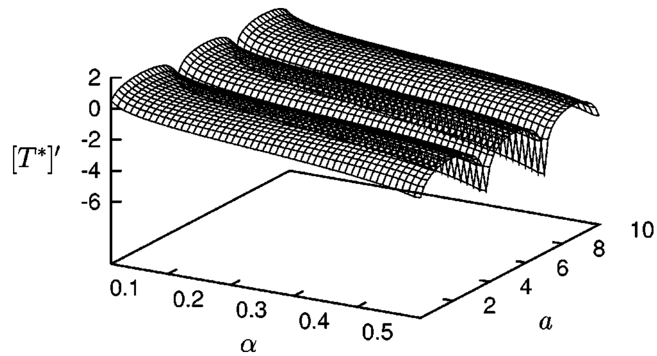

FIG. 3. Behavior of the transition temperature $T^*(a,b,\alpha)$ as a function of the dimensionless dissipation parameter α and of the ac field amplitude E. The figure is obtained by taking $b=2\mu E_0/\omega_0=1$. $[T^*]'$ stands for $\log_{10}[kT^*/\hbar\Delta_{eff}]$.

$$\frac{2\pi k_B T^*(\alpha)}{\hbar\Delta_{eff}(\alpha)}=-\frac{\Gamma(\alpha+u^*)}{u^*\Gamma(1-\alpha+u^*)}|J_N(a)|^{1/(1-\alpha)}, \tag{18}$$

where Δ_{eff} is the renormalized transition matrix element in the undriven case [1,14], u^* is the real (negative) solution of the equation

$$u^*[\Psi(\alpha+u^*)-\Psi(1-\alpha+u^*)]=1, \tag{19}$$

and $\Psi(z)$ is the digamma function, which is defined as $\Psi(z)=d\ \ln\Gamma(z)/dz$ [15].

From Eq. (19) we can see that the transition temperature $T^*(a,N,\alpha)$ changes not only with α, but also with the intensity of the dc-ac field. Figure 3 (obtained by considering $b=N=1$) depicts the dependence of the transition temperature $T^*(a,N,\alpha)$ on the dimensionless dissipation parameter α and on the intensity of the modified ac field $a=2\mu E/\omega_0$. We can see that T^* oscillates, which is a consequence of the fact that $J_N(a)$ is an oscillatory function. The transition temperature $T^*(a,N,\alpha)$ decreases when the modified ac field intensity a is stronger. The decreasing of the transition temperature $T^*(a,N,\alpha)$ when the intensity of the dc field $b=N$ becomes stronger is directly related to the fact that the amplitude of the Bessel functions amplitude decreases when their order becomes higher. Consequently, we can alter the transition temperature $T^*(\alpha)$ at which the transition from the coherent to the incoherent behavior T^* occurs or even destroy the coherence by changing appropriately the dc-ac field intensity.

In summary, we have studied the time evolution of the average population in a two-level system interacting with an Ohmic boson bath and driven by a dc-ac field. In the high-frequency driving case, we have derived the integro-differential master equation within the NIBA for the average transition probability. We have carried out the exact formal solution of the average population equation through Laplace transformers, obtaining the behavior of dissipative quantum decay for $\alpha > 1/2$. The external electric field was shown to be capable of destroying coherence and the temperature T^* at which the transition from a coherent to an incoherent behavior occurs depends on the driving field intensity.

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